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Optimal Station-Change Maneuver for Geostationary Satellites Using Constant Low Thrust

Nathan A. Titus*
Phillips Laboratory, Kirtland Air Force Base, New Mexico 87117

An electric propulsion system supplying constant low thrust can perform station-change maneuvers for geostationary orbits with significantly lower fuel costs than conventional propulsion systems. Using traditional optimal-control techniques, this paper develops a thrust-angle profile that improves upon the tangential-thrust profile originally suggested by Edelbaum for this problem. The optimal-control method ensures that the final orbit has zero eccentricity while achieving a rate of relocation that is essentially identical to the rate achieved by the tangential-thrust method. This reduction in eccentricity eliminates daily oscillations in the station of the relocated satellite. The proposed method generates the optimal-control profile using a spherical gravitational model with constant thrust and constant mass flow rate to describe the dynamics during the transfer. Simulations show that the eccentricity buildup depends on the level of acceleration, and that it can be reduced by up to three orders in magnitude using one common electric propulsion system. A comparison of station-change maneuvers using electric propulsion systems versus conventional chemical propulsion systems is included to emphasize the stationing flexibility that would be available to communications satellites using an electric propulsion system.

Nomenclature

a = semimajor axis
F = thruster force
f = state-vector derivative
g = gravitational acceleration
H = Hamiltonian
I_{sp} = specific impulse
J = performance index
m = satellite mass
$\dot{m}$ = mass flow rate
P = orbital period
r = radial position
T = total transfer time
t = time
u = radial velocity
v = transverse velocity
x = state vector $(r, u, v)^T$
θ = true longitude
λ = Lagrange multiplier vector
μ = Earth gravitational parameter (3.986012×10^5 km^3/s^2)
σ = satellite station (longitude)
ϕ = thrust-vector angles
$\omega_\oplus$ = Earth rotation rate (7.292116×10^{-5} rad/s)

Subscripts

0 = initial value
f = final value
geo = value for geostationary orbit
t = value for transfer orbit

Introduction

APPLICATIONS for constant low thrust have appeared throughout the literature since the 1960s, when practical electric propulsion systems were first demonstrated in the lab and in actual space operations.[1] Many of the studies concern the use of electric propulsion for geosynchronous satellite stationkeeping, which was the first application of an electrothermal thruster in space operations on the VELA nuclear detection satellite in 1965.[2] Later examples of stationkeeping using electric propulsion are the Applied Technology Satellites (ATS) and the Synchronous Meteorological Satellite (SMS-C).[3] The other significant area of study has been low-thrust orbit transfers, particularly for interplanetary and LEO-to-GEO transfers.[4–6] These include numerous papers discussing the use of optimal-control trajectories to minimize time or fuel usage of constant-thrust transfers.

One application that has not received extensive attention is orbit relocation maneuvers. These maneuvers are typically done following the initial on-orbit testing of the satellite but are also done to relocate on-orbit spares or to meet changing user demands. This application deserves some consideration, since the high specific impulse attainable with electric systems makes them an attractive option for decreasing the fuel cost of station changes. Communications satellites could then be relocated quickly without using a significant portion of the spacecraft's fuel budget. This would improve flexibility, increasing the competitiveness and efficiency of communications constellations.

One of the earliest authors to consider station changes using continuous low thrust was Edelbaum.[7] He suggested that the optimal method was to thrust tangentially in one direction until half of the desired change was completed and then thrust in the opposite direction until the full change was complete. In his development of this solution, he assumes that mass flow is negligible and he does not consider the eccentricity changes caused by tangential thrusting. His analysis also assumes a nearly circular orbit at all times; thus his use of the term "tangential" to describe thrust in the transverse direction is not inappropriate. However, in this paper the term "transverse" is used to more accurately describe the method suggested by Edelbaum. Isley and Duck also discuss simple station changes.[3] In their paper, they make the same assumptions as Edelbaum, but also allow a coast phase, creating a tradeoff between transfer time and fuel use.

While the zero-mass-flow assumption does not significantly affect the transfer dynamics, neglecting the eccentricity buildup of the transfer can be a disadvantage of the transverse control method. Many communications satellites in use today have strict stationkeeping requirements, and even a small eccentricity can cause daily periodic variations in the satellite's station. This paper presents a control method that maximizes the station change while constraining the final eccentricity to be zero, using well-known optimal-control techniques to determine the thrust-angle function.

Equations of Motion

In deriving the satellite's equations of motion, this paper assumes a simple spherical gravitational field with no perturbations due to

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*Captain, U.S. Air Force, Orbital Dynamics Program Manager, Space and Missiles Dynamics Division (PL/VTA). Member AIAA.

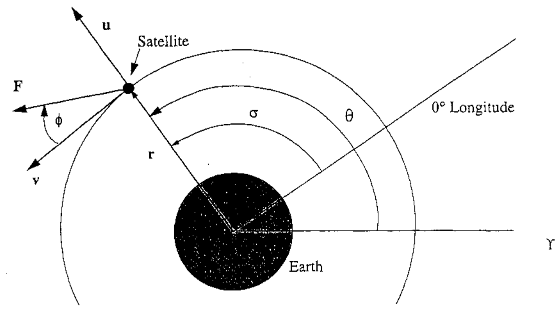

Fig. 1 Satellite motion.

atmospheric drag, solar radiation pressure, third-body effects, or other small forces. A constant thrust, acting in the orbital plane, provides the control input. Using a polar coordinate system, the equations of motion are

$$\begin{aligned} \dot{r} &= u \\ \dot{u} &= \frac{v^2}{r} - \frac{\mu}{r^2} + \frac{F \sin\phi}{m_0 - \dot{m}t} \\ \dot{v} &= -\frac{uv}{r} + \frac{F\cos\phi}{m_0 - \dot{m}t} \\ \dot{\theta} &= \frac{v}{r} \end{aligned} \tag{1}$$

An additional coordinate of interest is the longitude, or station, of the satellite. The differential equation describing its time rate of change is determined by the geometry of the problem, as shown in Fig. 1. It can be written as

$$\dot{\sigma} = \dot{\theta} - \omega_\oplus \tag{2}$$

Optimal-Control Formulation

In formulating the optimal-control problem, there are two paths to finding an optimal station-change trajectory. First, one can seek a minimum-time solution for a desired (fixed) station change. Alternatively, the problem can be framed as a fixed-time problem in which the optimal solution maximizes the station change. Although the first approach is more straightforward conceptually, the second approach requires less computational effort. In both cases, the resulting trajectory is the minimum-fuel trajectory under the assumption of continuous thrust during the transfer. While the mathematical proof is complex, a short example can demonstrate the equivalence of the two methods. Consider an optimal transfer of time t_A, where the maximum station change is computed by the second method to be σ_A. Now assume that the first method is used to compute the minimum time t_B, for a fixed station change of σ_A. If $t_B > t_A$, the minimum-time method has failed to produce a minimum, since the maximum-change method resulted in a transfer of equal station change in less time. Conversely, if $t_B < t_A$, then the maximum station change method has failed, since the minimum time method achieves a change of σ_A in time t_B, leaving time $t_A - t_B$ in which to make an additional station change in the same direction. The sum of the first change, σ_A, and the second (small, but finite) change must be greater than σ_A. Therefore, t_B must equal t_A, and the two methods are equivalent. The practical difference is that an operator is more likely to fix the station change than the transfer time when planning a transfer. This can be handled through the use of a graph like Fig. 2, with which the transfer time can easily be found when given the desired station change.

Optimization techniques based on variational calculus are well suited to solving this problem, and these techniques have been discussed extensively in the literature. For convenience, this paper finds the optimal trajectory using the second approach described above. Formally stated, the problem is to maximize the station change for a fixed transfer time where the initial and final state variables are constrained so that the satellite begins and ends in geostationary orbit. The single control input is the planar thrust angle. The formulation below leaves out many of the details, but closely follows the development of the theory given in Bryson and Ho.[8]

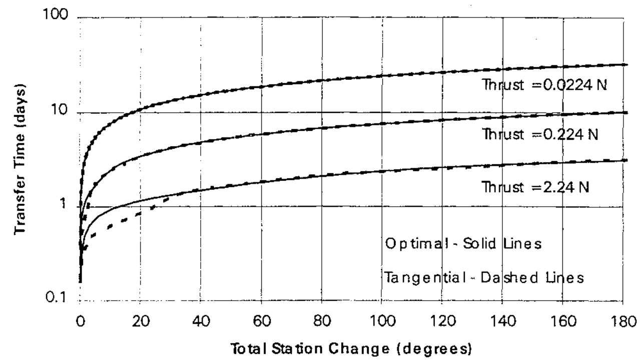

Fig. 2 Minimum transfer time for station change.

The performance index to be optimized is the station change. Maxima correspond to positive station-change trajectories, and minima correspond to negative station changes. The cost function can be expressed as the integral of the station-change rate over the total transfer,

$$J = \int_{t_0}^{t_f} \left(\frac{v}{r} - \omega_\oplus\right) dt \tag{3}$$

Next, the equations of motion, $\dot{x} = f(x, \phi, t)$, are adjoined to the performance index with Lagrange multiplier functions $\lambda(t)$:

$$\begin{aligned} J &= \int_{t_0}^{t_f} \left(\frac{v}{r} - \omega_\oplus\right) + \lambda^T[\dot{x} - f(x, \phi, t)]\, dt \\ &= \int_{t_0}^{t_f} (H + \lambda^T \dot{x})\, dt \end{aligned} \tag{4}$$

where the Hamiltonian H is defined as

$$H = \left(\frac{v}{r} - \omega_\oplus\right) - \lambda^T f \tag{5}$$

By setting the variation of J equal to zero, it can be shown that the performance index is optimized by the optimality condition

$$\frac{\partial H}{\partial \phi} = 0 \tag{6}$$

which leads to the control law

$$\tan\phi = \frac{\lambda_u}{\lambda_v} \tag{7}$$

where $\lambda_r, \lambda_u, \lambda_v$ are determined by the differential equations

$$\begin{aligned} \dot{\lambda}_r &= \frac{1}{r^2}\left[\lambda_u\left(v^2 - \frac{2\mu}{r}\right) - \lambda_v uv + v\right] \\ \dot{\lambda}_u &= \lambda_v \frac{v}{r} - \lambda_r \\ \dot{\lambda}_v &= \frac{1}{r}(\lambda_v u - 2\lambda_u v - 1) \end{aligned} \tag{8}$$

Combining the first three state equations from Eq. (1) with the costate equations (8) and the control law from Eq. (7), there are six first-order nonlinear ordinary differential equations (ODEs) that describe the optimal trajectory. Since the dynamics do not depend on the true longitude, the equations for θ and λ_θ can be dropped to decrease the computational load. In order to define this system exactly, six boundary conditions are also needed. These are the initial and final values for r, u, and v, which are

$$\begin{aligned} r(t_0) &= r(t_f) = r_{\text{geo}} = 42164.2\,\text{km} \\ u(t_0) &= u(t_f) = u_{\text{geo}} = 3.07466\,\text{km/s} \\ v(t_0) &= v(t_f) = v_{\text{geo}} = 0.0\,\text{km/s} \end{aligned} \tag{9}$$

The system can be solved numerically using the "shooting" method for boundary-value problems.[9] Using this method, the initial values for λ_r, λ_u, λ_v are guessed and then the ODEs are integrated numerically to the final time. Next, the errors in the final conditions of r, u, and v are computed. Then a Jacobian matrix (relating small changes in the initial λ to small changes in the final states) is created numerically. Using this matrix, new guesses for λ_r, λ_u, λ_v are made, and the process is repeated until the errors in the final states are acceptable.

In the formulation given above, there is no differentiation between positive and negative station changes. In practice, the numerical solution method is as likely to converge to one as the other, given random guesses for the initial values of the costates. Experience showed that using initial guesses of $\lambda_u = \pm t_f$ and $\lambda_r, \lambda_v = \pm 10 t_f$ where t_f is measured in days, usually converged fairly rapidly to a solution. The positive signs resulted in minima of the cost function (negative station changes), and the negative signs resulted in maxima (positive station changes). This rule of thumb broke down occasionally, and for very short transfer times ($t_f < 1$) it did not work. In these cases, using converged solutions from nearby transfer times as the initial guess generally gave good results.

Evaluation of the Optimal Station Change

The performance of the optimal-control method was evaluated using simulations to predict the behavior of a satellite during a station-change maneuver, first using optimal control, and then using the transverse thrust suggested by Edelbaum. Simulation results are shown only for positive station changes, since the results for negative station changes are very similar and offer little additional insight. The dynamics were modeled using the equations of motion (1) and propagated with a fourth-order Runge–Kutta integration scheme. The constants in the equations were chosen to represent a typical communications satellite and typical form of electric propulsion. Specifically, the following figures were developed assuming a geostationary satellite with an initial mass of 1000 kg and a propulsion system using an arcjet with a specific impulse of 1000 s. The total thrust would depend primarily on the power available, so various thrust levels for the arcjet were considered: 2.24, 0.224, and 0.0224 N. These levels correspond to an initial acceleration of 0.01, 0.001, and 0.0001 in a unit system where a geosynchronous orbit has a radius of one distance unit and a velocity of one distance unit per time unit, and one mass unit is 1000 kg.

Figure 2 compares the minimum transfer time for a station change using each method. The results for the transverse case agree well with those given in Isley and Duck,[3] which indicates that the mass loss during the transfer is indeed largely negligible. It is also evident from the figure that the two methods produce very similar results. For the lowest-thrust case, the two methods are virtually indistinguishable, but an examination of the highest-thrust case reveals a slight periodic variation in which the transverse control method is faster than the optimal control method for some values of the total station change. This apparent anomaly results from the absence of constraints on many of the final orbital elements when applying the transverse method. Whereas the optimal method forces the satellite back into a perfectly geostationary orbit following a transfer, the transverse method guarantees only a return to a geosynchronous period. The additional cost of restoring the orbit to zero eccentricity causes the optimal solution to be slower than the transverse solution in some cases.

Evidence for the effect of the eccentricity buildup caused by the transverse method is found in Fig. 3, which shows the residual eccentricity as a function of the transfer time. (Using constant thrust, there is a one-to-one mapping of the total station change to the total transfer time. In Figs. 3–5, total transfer time, rather than total station change, was chosen as the independent variable used to describe transfers of different length.) For the transverse method, the final eccentricity reaches a maximum when the duration of the transfer spans an odd number of revolutions and a minimum when it spans an even number of revolutions. The quasiperiodic behavior of the transverse curve is related to the average period for two revolutions of the satellite during a station-change maneuver. This cycle is very close to 2 days for short transfers, but for long transfers the maneuver will cause larger changes in the semimajor axis before returning to geosynchronous altitude, and as a result the average time for two revolutions will differ noticeably from the 2-day cycle. The cyclic behavior correlates directly with the variations seen in Fig. 2. Transfers spanning an even number of revolutions require little energy to recircularize, so the optimal method outperforms the transverse method in these cases, whereas transfers spanning an odd number of revolutions build up a relatively large eccentricity, so that the optimal method must divert energy from the station change to the recircularization of the final orbit. This "cost of recircularization" phenomenon is also seen in orbit-raising problems using constant thrust, and is discussed in Alfano and Thorne.[5]

The eccentricity buildup is due to the continuous thrust and can be explained by considering two positive station changes using transverse control—first a transfer spanning two revolutions, and second a transfer spanning a single revolution. In the two-revolution transfer, the satellite begins by thrusting opposite the direction of motion. Throughout the first half of one revolution, the flight-path angle is negative and the transverse thrusting will act to raise the orbital eccentricity. From the one-half-revolution point to the one-revolution point, the flight-path angle is positive and the transverse thrusting acts to decrease eccentricity, so that after one full revolution, the satellite has zero eccentricity again. Then the thrust direction is reversed, and now the positive flight-path angle causes an eccentricity increase during the first half revolution, whereas the negative angle causes a decrease in the second half revolution. After a total of two revolutions, the station change is complete and there is no residual eccentricity. Now consider a maneuver that spans one revolution. During the first half revolution, the satellite builds up eccentricity as before, but now the thrust reversal occurs at the half-revolution point. Consequently, the eccentricity continues to build in the second half revolution, resulting in the maximum residual eccentricity

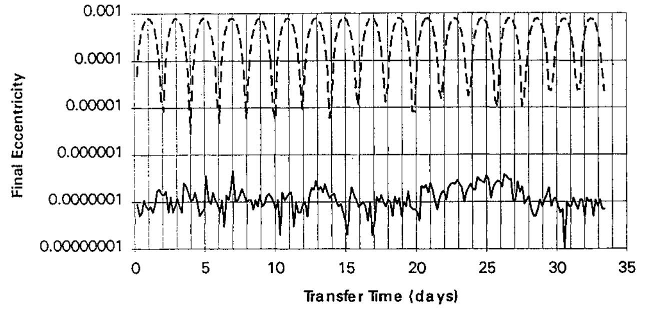

Fig. 3 Eccentricity buildup due to station-change maneuver: —— optimal and - - - - tangential.

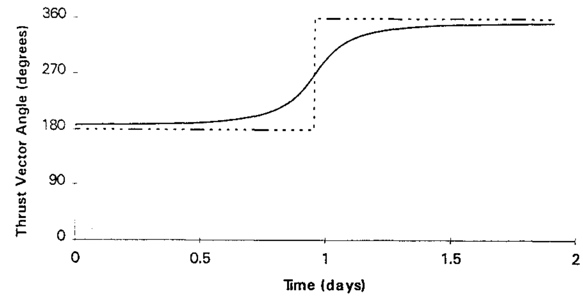

Fig. 4 Long-transfer control profile: —— optimal and - - - - tangential.

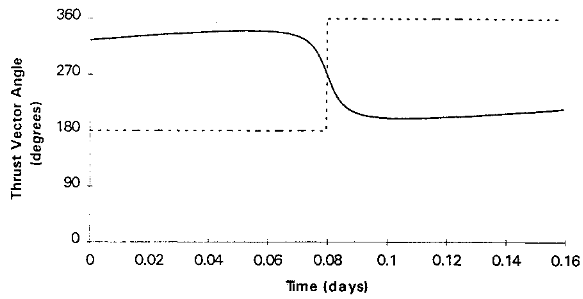

Fig. 5 Short-transfer control profile: —— optimal and - - - - tangential.

for a given thrust level. For transfers that span a nonintegral number of revolutions, the residual eccentricity lies between zero and this maximum.

The eccentricity buildup caused by the transverse method can have significant ramifications for the successful operation of a communications satellite. The residual eccentricity shown in Fig. 3 was generated using the lowest-thrust (0.0224 N) family of transfers. The residual eccentricity resulting from the transverse method is, on average, about 3 orders of magnitude greater than the final eccentricity resulting from the optimal method. (By design, the optimal method should produce zero final eccentricity, but it is actually a function of the numerical accuracy of the method and machine used to compute the optimal control function. In these simulations, the final values of r, u, and v were computed to six significant figures, so the final eccentricities are zero only to six significant figures.) Consider the worst case from the figure, $e = 0.001$. Using simple calculations, it can be shown that a geosynchronous satellite with this eccentricity would have a daily station oscillation of about ± 0.11 deg. Many communications satellites, such as INTELSAT-V and DSCS III,[10] have station tolerance requirements of ± 0.1 deg, so even a small eccentricity can cause the satellite to exceed its station tolerances. Thrust levels higher than the 0.0224 N used in generating Fig. 3 would result in even higher maximum eccentricities.

The similarity in station-change rates shown in Fig. 2 implies that the control profile of the transverse method is already nearly optimal, and that the method presented in this paper is only a slight variation. While this is true for most transfers, for very short transfers the optimal-control profile is actually quite different from the transverse-control profile. Figure 4 shows a 2-day transfer for both the transverse and optimal profiles. In this case, the transverse profile is a reasonable approximation of the optimal. By initially thrusting in a direction nearly opposite the velocity, the satellite undergoes a transient motion in the wrong direction, but it then loses altitude and shortens its period. This results in forward motion relative to the geosynchronous orbit. For a short transfer time, the optimal profile is nearly the opposite of the transverse profile. Figure 5 shows the profile for a transfer spanning only one-sixth of a day. A short maneuver of this type is more appropriately known as east–west stationkeeping, rather than station changing. In this maneuver, the initial thrusting in the direction of the velocity moves the satellite forward, but the second half of the burn restores the satellite to a geosynchronous orbit before it can be affected by the altitude loss and associated period change. The transverse profile used for the long transfer does not provide a decent approximation for these maneuvers, whereas the optimal-control method works equally well for both transfer lengths. This suggests that an electric propulsion system could perform both functions with the same optimal-control algorithm, simplifying the overall control architecture.

Comparison of High- and Low-Thrust Maneuvers

Having established the best specific method of using low thrust, it may be beneficial to review the argument for low thrust in general. Electric propulsion has been widely accepted as offering large fuel savings when compared to conventional methods, and the following simple derivation quantifies these benefits in the problem of orbit relocation, by comparing the approximate fuel costs of continuous-thrust electric propulsion systems with that of conventional hydrazine propulsion systems. For satellites with hydrazine thrusters, the typical (and most efficient) relocation maneuver consists of two impulsive burns. The first burn establishes a drift orbit, and the second burn stops the drift and recircularizes the orbit. By simply waiting for the satellite to drift to the new location, any size of station change can be achieved with very little fuel. However, the drift rate is directly proportional to the ΔV used. Therefore an unbiased metric for the comparison of the two-burn and continuous-thrust methods is the cost associated with producing equal average relocation rates.

For the two-burn transfer, the relocation rate $\dot{\sigma}$ depends on the difference of the angular velocity of the geosynchronous and transfer orbits,

$$\dot{\sigma} = \omega_t - \omega_\oplus \tag{10}$$

In terms of the orbital periods, this is

$$\dot{\sigma} = \frac{2\pi}{P_t} - \frac{2\pi}{P_{\text{geo}}} = \omega_\oplus\left(\frac{P_{\text{geo}}}{P_t} - 1\right) \tag{11}$$

This expression can then be written using the semimajor axis of each orbit:

$$\dot{\sigma} = \omega_\oplus\left(\frac{1}{(1 + \Delta a/a_{\text{geo}})^{\frac{3}{2}}} - 1\right) \tag{12}$$

Using a binomial series expansion and dropping all terms higher than first order, this can be approximated as

$$\dot{\sigma} = -\frac{3}{2}\frac{\Delta a}{a_{\text{geo}}}\omega_\oplus \tag{13}$$

The semimajor-axis change is related to ΔV through a simple approximation obtained by differentiating the energy equation while holding the radius constant,

$$\Delta a = \frac{2 v_{\text{geo}} a_{\text{geo}}^2}{\mu}\Delta V \tag{14}$$

Substituting this result into Eq. (13) gives the relationship

$$\Delta V = -\frac{r_{\text{geo}}}{3}\dot{\sigma} \tag{15}$$

This equation gives the ΔV needed to generate a given drift rate. However, for a useful transfer, a stopping burn is also required, so the total ΔV for the two-burn transfer is twice this expression, or

$$\Delta V_{\text{two-burn}} = -\tfrac{2}{3} r_{\text{geo}}\dot{\sigma} \tag{16}$$

Assuming a long transfer (one that occurs over several days or more), the rate $\dot{\sigma}$ is the average rate, since it is effectively constant throughout the transfer.

For the continuous-thrust case, determining the appropriate drift rate is slightly more complex, since it cannot be considered constant during the transfer. One approach is to integrate the drift rate over the entire transfer to compute the total station change, and then divide by the transfer time to obtain an average drift rate. Start with Eq. (15), and substitute $(F\,\Delta t)/m_0$ for ΔV, giving

$$\dot{\sigma} = -\frac{3F\,\Delta t}{a_{\text{geo}} m_0} \tag{17}$$

If the semimajor axis is assumed to be approximately constant during the transfer (which is reasonable for drift rates up to about 30 deg/day), then this equation can be integrated to obtain

$$|\sigma| = \frac{3}{2}\frac{|F|}{a_{\text{geo}} m_0}T^2 \tag{18}$$

This expression assumes that the thrust is constant, but in reality the transfer will involve thrusting in one direction for half of the transfer time, and then thrusting in the opposite direction for the second half of the transfer. This is essentially a symmetric process, so the total station change is twice the station change achieved in the first half of the transfer. In the equation this becomes

$$\sigma = 2\left[\frac{3}{2}\frac{|F|}{a_{\text{geo}} m_0}\left(\frac{T}{2}\right)^2\right] = \frac{3}{4}\frac{|F|}{a_{\text{geo}} m_0}T^2 \tag{19}$$

Substituting $\dot{\sigma}_{\text{average}} = \sigma/T$ and $\Delta V_{\text{continuous}} = FT/m_0$, this becomes

$$|\Delta V|_{\text{continuous}} = \tfrac{4}{3} r_{\text{geo}} |\dot{\sigma}|_{\text{average}} \tag{20}$$

A comparison of Eqs. (16) and (20) shows that a high-thrust conventional propulsion system is twice as efficient as an electric propulsion system in terms of ΔV. The advantage of electric propulsion is only seen when the ΔV is written in terms of the fuel used. The impulse can be approximated by

$$\Delta V \approx \frac{F\,\Delta t}{m_0} = \frac{I_{sp} g \dot{m}\,\Delta t}{m_0} = \frac{I_{sp} g\,\Delta m}{m_0} \tag{21}$$

Substituting this expression for ΔV into Eqs. (16) and (20) and solving for Δm (fuel used) gives

$$\Delta m_{\text{two-burn}} = \frac{2 r_{\text{geo}} m_0}{3 I_{sp} g} \dot{\sigma}_{\text{average}} \tag{22}$$

$$\Delta m_{\text{continuous}} = \frac{4 r_{\text{geo}} m_0}{3 I_{sp} g} \dot{\sigma}_{\text{average}} \tag{23}$$

These expressions clarify the importance of the specific impulse of the propulsion system. On improving the specific impulse of the continuous system by a factor of 2, the advantage of the high-thrust system is eliminated, and the improvement factor of electric systems is generally much greater than 2. For example,[11] the most common type of propulsion system today uses monopropellant hydrazine with a specific impulse of about 200–225 s, while a typical arcjet can have a specific impulse of more than 1000 s. The inefficiency of continuous-thrust maneuvers compared to impulsive maneuvers is overshadowed by the efficiency of the low-thrust propulsion systems compared to hydrazine propulsion systems, clearly demonstrating the value of low-thrust propulsion for use on satellites for a variety of tasks.

Conclusion

The application of optimal-control techniques to the problem of relocating satellites in geostationary orbit with constant low thrust has a significant advantage over the simple transverse-thrust control method. The optimal method ensures the new orbit has zero eccentricity, while achieving a rate of relocation that is essentially identical to transverse thrust. By constraining the final eccentricity, significant daily variations in the station are avoided. This can be an important factor in minimizing stationkeeping effort following a station-change maneuver.

Constant low thrust should be used to perform station-change maneuvers on future geosynchronous satellite systems. The simple derivation presented here highlights the fuel savings achievable by low-thrust electric propulsion systems compared with conventional propulsion systems. This reduction in the fuel costs of station changes could translate to longer life and more frequent or more rapid station changes. Ultimately this allows greater flexibility and improved overall performance of our worldwide communications systems.

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